NAG Toolbox for MATLAB

g01gc

1 Purpose

g01gc returns the probability associated with the lower tail of the noncentral χ^2-distribution via the function name.

2 Syntax

```
[result, ifail] = g01gc(x, df, rlamda, tol, 'maxit', maxit)
```

3 Description

The lower tail probability of the noncentral χ^2-distribution with ν degrees of freedom and noncentrality parameter λ, $P(\mathbf{x} \leq x : \nu;\lambda)$, is defined by

$$P(X \leq x : \nu;\lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2}\frac{(\lambda/2)^j}{j!}P(X \leq x : \nu + 2j;0), \tag{1}$$

where $P(\mathbf{x} \leq x : \nu + 2j;0)$ is a central χ^2-distribution with $\nu + 2j$ degrees of freedom.

The value of j at which the Poisson weight, $e^{-\lambda/2}\frac{(\lambda/2)^j}{j!}$, is greatest is determined and the summation (1) is made forward and backward from that value of j.

The recursive relationship:

$$P(X \leq x : a + 2;0) = P(X \leq x : a;0) - \frac{(x^a/2)e^{-x/2}}{\Gamma(a+1)} \tag{2}$$

is used during the summation in (1).

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The deviate from the noncentral χ^2-distribution with ν degrees of freedom and noncentrality parameter λ.

Constraint: $\mathbf{x} \geq 0.0$.

2: **df – double scalar**

ν, the degrees of freedom of the noncentral χ^2-distribution.

Constraint: $\mathbf{df} \geq 0.0$.

3: **rlamda – double scalar**

λ, the noncentrality parameter of the noncentral χ^2-distribution.

Constraint: **rlamda** ≥ 0.0 if **df** > 0.0 or **rlamda** > 0.0 if **df** $= 0.0$.

4: **tol – double scalar**

The required accuracy of the solution. If g01gc is entered with **tol** greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see x02aj), then the value of $10 \times$ ***machine precision*** is used instead.

5.2 Optional Input Parameters

1: **maxit – int32 scalar**

The maximum number of iterations to be performed.

Suggested value: 100. See Section 8 for further discussion.

Default: 100

Constraint: **maxit** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01gc may return useful information for one or more of the following detected errors or warnings.

If on exit **ifail** $= 1$, 2, 4 or 5, then g01gc returns 0.0.

ifail $= 1$

On entry, **df** < 0.0,
or **rlamda** < 0.0,
or **df** $= 0.0$ and **rlamda** $= 0.0$,
or **x** < 0.0,
or **maxit** < 1.

ifail $= 2$

The initial value of the Poisson weight used in the summation (1) was too small to be calculated. The value of $P(\mathbf{x} \leq x : \nu;\lambda)$ is likely to be zero.

ifail $= 3$

The solution has failed to converge in **maxit** iterations.

ifail $= 4$

The value of a term required in (2) is too large to be evaluated accurately. The most likely cause of this error is both **x** and **rlamda** being very large.

ifail = 5

The calculations for the central χ^2 probability has failed to converge. This is an unlikely error exit. A larger value of **tol** should be used.

7 Accuracy

The summations described in Section 3 are made until an upper bound on the truncation error relative to the current summation value is less than **tol**.

8 Further Comments

The number of terms in (1) required for a given accuracy will depend on the following factors:

(i) The rate at which the Poisson weights tend to zero. This will be slower for larger values of λ.

(ii) The rate at which the central χ^2 probabilities tend to zero. This will be slower for larger values of ν and x.

9 Example

```
x = 8.26;
df = 20;
rlamda = 3.5;
tol = 5e-06;
[result, ifail] = g01gc(x, df, rlamda, tol)
```

```
result =
    0.0032
ifail =
           0
```